HW3A. Written Homework 3A. **Due Week 3 Wednesday 11:59PM**

Name:

Instructions: Upload a pdf of your submission to **Gradescope**. This worksheet is worth 20 points: up to 8 points will be awarded for accuracy of certain parts (to be determined after the due date) and up to 12 points will be awarded for completion of parts not graded by accuracy.

(1) Determine if the following sets of vectors are **linearly independent** or **linearly dependent**. Show supporting work (i.e. by what method did you determine independence/dependence?)

(a) $V = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -4 \\ 4 \end{pmatrix} \right\}$

(b) $W = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$

(c) $A = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, \begin{pmatrix} 5 \\ 7 \\ 9 \end{pmatrix} \right\}$

(2) For each given $\mathbf{N} \in \mathbb{R}^3$: Find two vectors $\mathbf{d_1}, \mathbf{d_2} \in \mathbb{R}^3$ such that $\mathbf{d_1}$ and $\mathbf{d_2}$ are both orthogonal to $\mathbf{N}$ and the set $\{\mathbf{N}, \mathbf{d_1}, \mathbf{d_2}\}$ is a basis for $\mathbb{R}^3$. There are multiple possible answers here.

(a) $\mathbf{N} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

(b) $\mathbf{N} = \begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}$

(c) $\mathbf{N} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$

(3) For each given vector $\mathbf{v} \in \mathbb{R}^2$, find another vector $\mathbf{w} \in \mathbb{R}^2$ such that $\mathbf{v}$ is orthogonal to $\mathbf{w}$ and the set $\{\mathbf{v}, \mathbf{w}\}$ is a basis for $\mathbb{R}^2$. Note that there are multiple possible answers.

(a) $\mathbf{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(b) $\mathbf{v} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

(c) $\mathbf{v} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

(4) For each of the following set of vectors V, choose some subset W of V such that (1) W is linearly independent and (2) $\operatorname{span} W = \operatorname{span} V$. Note there may be multiple possible answers for W for each item.

(a) $V = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -4 \\ 4 \end{pmatrix} \right\}$

(b) $V = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, \begin{pmatrix} 7 \\ 8 \\ 10 \end{pmatrix} \right\}$

(c) $V = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ -4 \end{pmatrix} \right\}$

(d) $V = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \right\}$